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Z-contraction condition involving simulation function in b-metric space under fixed points considerations

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Abstract. The purpose of this paper is to prove a common fixed point theorems for two pairs of mappings under the generalized Z-contraction with respect to the concept of simulation function in b-metric space. Our paper generalizes some fixed point theorems in literature [6, 13, 16, 18].

1. Introduction

In the Brower's Fixed Point Theorem, the function f does not give any unique fixed point. In the history of fixed point theorem and applications, it was Banach [8] who introduced the concept of contraction condition in the year 1922 for obtaining fixed as well as unique fixed point. After Banach's remarkable result good number of Fixed Point Theorist started working in the area of developing theory in the lines of Banach. One can refer Rhoades [Trans. AMS, 1977] for various types of contraction as well as non-contraction type conditions which facilitates the contraction map to get unique fixed point. The proof of the theorem is simple and elegant for obtaining fixed points. Later after 50's authors tried to replace the metric space by Menger spaces, Quasi metric spaces, Fuzzy metric, b-metric spaces, etc.

Bakhtin [7] and Czerwik [9] generalized the concept of metric type space by introducing b-metric spaces.

On the other hand, Aamri and Moutawakil in [1] introduced the idea of $(E.A)$-property in metric spaces. Moreover many authors proved some fixed point theorems for single-valued using this concept in metric spaces, readers may consult [2, 5, 12, 15, 17, 19–22] as a reference.

Motivated by the results and notions mentioned above. In this paper, we present generalized Z-contractions involving simulation functions and

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establish several common fixed point theorems for this class of mappings defined on b-metric spaces. Our main result is essentially extended and generalized the results of [6].

2. Preliminaries

The following notions are necessary to establish a fixed point theorem in this paper.

Definition 1. Let X be a (nonempty) set and $s > 1$ be a given real number. A function $d : X \times X \to [0, \infty)$ is a b-metric on X if for all $x, y, z \in X$, the following conditions hold:

(b1) $d(x, y) = 0$ if and only if $x = y$,
(b2) $d(x, y) = d(y, x)$,
(b3) $d(x, z) \le s[d(x, y) + d(y, z)]$.

A triplet (X, d, s) is called a b-metric space.

Also, every metric space is a b-metric space but the converse is not necessarily true.

Example 1. The triplet $([0, 1], d, 2)$, where $d : X \times X \to [0, \infty[$ is given by $d(x; y) = |x - y|^2$ for all $x, y \in X$, is a 2-metric space; but it is not a metric space.

Example 2 ([3])**.** Let (X, d) be a metric space and $\rho(x, y) = (d(x, y))^p$, where $p > 1$ is a real number. Then ρ is a b-metric with $s = 2^{p-1}$.

Definition 2 ([10])**.** Let (X, d) be a b-metric space and $\{x_n\}$ a sequence in X. The sequence $\{x_n\}$ is said to be

(i) convergent to $x \in X$ if $\lim\limits_{n\to\infty} d(x_n, x) = 0$. In this case, we write $\lim\limits_{n\to\infty} x_n = x$;
(ii) a Cauchy sequence if $\lim\limits_{n,m\to\infty} d(x_n, x_m) = 0$;
(iii) (X, d) is complete if every Cauchy sequence in X is convergent.

Proposition 1. *In a b-metric space (X, d), the following assertions hold.*

(p_1) *A convergent sequence has a unique limit.*
(p_2) *Each convergent sequence is a Cauchy sequence.*
(p_3) *In general, a b-metric is not continuous in each variable, see [4, 11].*

Definition 3 ([19])**.** Let (X, d) be a b-metric space and f and g be selfmappings on X.

i) f and g are said to compatible if whenever a sequence $\{x_n\}$ in X is such that fx_n and gx_n are b-convergent to some $t \in X$, then

$$\lim_{n\to\infty} d\left(fgx_n, gfx_n\right) = 0.$$

ii) f and g are said to noncompatible if there exists at least one sequence $\{x_n\}$ in X is such that fx_n and gx_n are b-convergent to some $t \in X$, but $\lim\limits_{n\to\infty} d(fgx_n, gfx_n)$ is either nonzero or does not exist.

iii) f and g are said to satisfy the b-$(E.A)$ property if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} gx_n = fx_n = t.$$

Definition 4 ([14])**.** f and g be given self-mappings on a set X. The pair f, g is said to be weakly compatible if f and g commute at their coincidence points

In 2015, Khojasteh, Shukla and Radenovic [16] introduced simulation functions and defined Z-contraction with respect to a simulation function and it includes a large class of contractive conditions which follows:

Definition 5 ([16])**.** A simulation function is a mapping $\zeta : [0,\infty)\times[0,\infty) \to (-\infty,\infty)$ satisfying the following conditions:

ζ1) $\zeta(0,0) = 0$;

ζ2) $\zeta(t,s) < s - t$ for all $s,t > 0$.

The following are examples of simulation functions.

Example 3. Let $\zeta : [0,\infty) \times [0,\infty) \to (-\infty,\infty)$ be defined by

1. $\zeta(s,t) = \dfrac{s}{1+s} - t$, for all s,t in $[0,\infty)$.
2. $\zeta(t,s) = \lambda s - t$, for all s,t in $[0,\infty)$, where $\lambda \in [0,1)$.
3. $\zeta(t,s) = s - \lambda t$, for all s,t in $[0,\infty)$, where $\lambda > 1$.
4. $\zeta(t,s) = \dfrac{1}{s+1} - (t+1)$, for all $t,s \in [0,\infty)$.
5. $\zeta(t,s) = \dfrac{s}{s+1} - te^t$, for all $t,s \in [0,\infty)$.

Definition 6 ([16])**.** Let (X,d) be a metric space and $f : X \to X$ be a self map of X. We say that f is a Z-contraction with respect to ζ, if there exists a simulation function ζ such that

$$\zeta\left(d(fx,fy), d(x,y)\right) \geq 0 \text{ for all } x,y \in X.$$

In [6], Babu, Dubla and Kumar introduced and proved the generalized Z-contraction pair of maps with respect to ζ in the following way.

Definition 7. Let (X,d) be a b-metric space with coefficient $s > 1$. Let $f,g : X \to X$ be two self mappings. If there exists a simulation function ζ such that

$$\zeta\left(s^4 d(fx,gy), M(x,y)\right) \geq 0 \text{ for all } x,y \in X,$$

where

$$M(x,y) = \max\left\{d(x,y), d(x,fx), d(y,gy), \frac{d(x,gy)+d(fx,y)}{2s}\right\},$$

for all $x, y \in X$; then we say that (f, g) is a generalized Z-contraction pair of maps.

3. Main Result

Now, we introduce a generalized Z-contraction for two pair of maps with respect to ζ which follows:

Theorem 1. *Let $(X\ d)$ be a b-metric space with coefficient $s > 1$, and $f, g, S, T : X \to X$ be mappings with $f(X) \subset T(X), g(X) \subset S(X)$ such that*

$$\zeta(s^4 d(fx, gy), M(x, y)) \geq 0, \tag{1}$$

where

$$M(x,y) = \max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{d(fx,Ty)+d(Sx,gy)}{2s}\right\}.$$

Suppose that one of the pairs (f, S) and (g, T) satisfy the b-(E.A)-property and that one of the subspaces $f(X)$, $g(X)$, $S(X)$ and $T(X)$ is closed in X.

Then the pairs (f, S) and (g, T) have a point of coincidence in X. Moreover, if the pairs (f, S) and (g, T) are weakly compatible, then f, g, S and T have a unique common fixed point.

Proof. If the pairs (f, S) satisfies the b-$(E.A)$-property, then there exists a sequence $\{x_n\}$ in X satisfying

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = q,$$

for some $q \in X$. As $f(X) \subset T(X)$ there exists a sequence $\{y_n\}$ in X such that $fx_n = Ty_n$. Hence $\lim\limits_{n\to\infty} Ty_n = q$: Let us show that $\lim\limits_{n\to\infty} gy_n = q$.

$$\zeta(s^4 d(fx_n, gy_n), M(x_n, y_n)) \geq 0, \tag{2}$$

where

$$M(x_n,y_n) = \max\left\{\begin{array}{c} d(Sx_n,Ty_n), d(fx_n,Sx_n), d(gy_n,Ty_n), \\ \frac{d(fx_n,Ty_n)+d(Sx_n,gy_n)}{2s} \end{array}\right\}.$$

In (2), taking limit,

$$\zeta(s^4 \lim_{n\to\infty} d(q, gy_n), \lim_{n\to\infty} M(x_n, y_n)) \geq 0,$$

where

$$\begin{aligned} \lim_{n\to\infty} M(x_n,y_n) &= \max\left\{\begin{array}{c} d(q,q), d(q,q), \\ \lim\limits_{n\to\infty} d(gy_n,q), \lim\limits_{n\to\infty}\left(\frac{d(q,q)+d(q,gy_n)}{2s}\right) \end{array}\right\} \\ &= \lim_{n\to\infty} d(gy_n,q). \end{aligned}$$

So,

$$0 \leq \zeta(s^4 \lim_{n\to\infty} d(q, gy_n), \lim_{n\to\infty} d(gy_n, q)) < \lim_{n\to\infty} d(gy_n, q)\left(1 - s^4\right) \leq 0,$$

thus, $\lim_{n\to\infty} d(gy_n, q)$, hence $\lim_{n\to\infty} gy_n = q$.

We conclude that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} gy_n = q.$$

If $T(X)$ is closed subspace of X, then there exists a $r \in X$ such that

$$Tr = q. \tag{3}$$

By (1),

$$\zeta(s^4 d(fx_n, gr), M(x_n, r)) \geq 0,$$

where

$$M(x_n, r) = \max\left\{d(Sx_n, q), d(fx_n, Sx_n), d(gr, q), \frac{d(fx_n, q) + d(Sx_n, gr)}{2s}\right\}.$$

Letting $n \to \infty$,

$$\lim_{n\to\infty} \zeta(s^4 d(fx_n, gr), M(x_n, r)) \geq 0,$$

where

$$\begin{aligned} \lim_{n\to\infty} M(x_n, r) &= \max\left\{\begin{array}{c} \lim_{n\to\infty} d(Sx_n, q), \lim_{n\to\infty} d(fx_n, Sx_n), d(gr, q), \\ \lim_{n\to\infty}\left(\frac{d(fx_n,q)+d(Sx_n,gr)}{2s}\right) \end{array}\right\} \\ &= \max\left\{d(q, q), d(fq, q), d(gr, q), \left(\frac{d(q, q) + d(q, gr)}{2s}\right)\right\} \\ &= d(gr, q). \end{aligned}$$

Then

$$0 \leq \zeta(s^4 d(q, gr), d(gr, q)) < d(gr, q)\left(1 - s^4\right) \leq 0,$$

thus,

$$gr = q. \tag{4}$$

By (3) and (4) hold that the pair (g, T) have a coincidence point.

As $g(X) \subset S(X)$, there exists a point $z \in X$ such that $q = Sz$. We claim that $Sz = fz$. By (2), we have

$$\zeta(s^4 d(fz, q), M(z, r)) \geq 0,$$

where

$$\begin{aligned} M(z, r) &= \max\left\{d(Sz, q), d(fz, Sz), d(gr, q), \frac{d(fz, q) + d(Sz, gr)}{2s}\right\} \\ &= d(fz, q). \end{aligned}$$

Then

$$0 \leq \zeta(s^4 d(fz, q), d(fz, q)) < d(fz, q)\left(1 - s^4\right) \leq 0,$$

thus,

$$fz = Sz = q.$$

Hence z is a coincidence point of the pair (f,S). Thus $fz = Sz = gr = Tr = q$. By weak compatibility of the pairs (f,S) and (g,T), we deduce that $fq = Sq$ and $gq = Tq$.

We will show that q is a common fixed point of f, g, S and T.

From (1)

$$\zeta(s^4 d(fq,q), M(q,r)) = \zeta(s^4 d(fq,gr), M(q,r)) \geq 0, \tag{5}$$

where

$$\begin{aligned} M(q,r) &= \max\left\{d(Sq,Tr), d(fq,Sq), d(gr,Tr), \frac{d(fq,Tr)+d(Sq,gr)}{2s}\right\} \\ &= \max\left\{d(fq,q), d(fq,fq), d(q,q), \frac{d(fq,q)+d(fq,q)}{2s}\right\} \\ &= d(fq,q). \end{aligned}$$

By (5)

$$0 \leq \zeta(s^4 d(fq,q), d(fq,q)) = d(fq,q)\left(1-s^4\right) \leq 0,$$

so,

$$fq = Sq = q.$$

Similarly, it can be shown $gq = Tq = q$.

To prove the uniqueness of the fixed point of f, g, S and T. Suppose for contradiction that p is another fixed point of f, g, S and T. By (1), we obtain

$$\zeta(s^4 d(p,q), M(p,q)) = \zeta(s^4 d(fp,gq), M(p,q)) \geq 0,$$

where

$$\begin{aligned} M(p,q) &= \max\left\{d(Sp,Tq), d(fp,Sp), d(gq,Tq), \frac{d(fp,Tq)+d(Sp,gq)}{2s}\right\} \\ &= \max\left\{d(p,q), d(p,p), d(q,q), \frac{d(p,q)+d(p,q)}{2s}\right\} \\ &= d(p,q). \end{aligned}$$

Hence we have

$$0 \leq \zeta(s^4 d(p,q), M(p,q)) = d(p,q)\left(1-s^4\right) \leq 0,$$

which implies that $q = q$. □

Corollary 1. *Let (X,d) be a b-metric space with coefficient $s > 1$, and $f, g, S, T : X \to X$ be mappings with $f(X)$ subset $T(X)$, $g(X)$ subset $S(X)$. Suppose that there exists $\lambda \in [0,1)$ such that*

$$s^4 d(fx,gy) \leq \lambda M(x,y),$$

where

$$M(x,y) = \max\left\{d(Sx,Ty), d(fx,Sx), d(gy,Ty), \frac{d(fx,Ty)+d(Sx,gy)}{2s}\right\}.$$

Suppose that one of the pairs (f, S) and (g, T) satisfy the b-$(E.A)$-property and that one of the subspaces $f(X), g(X), S(X)$ and $T(X)$ is closed in X. Then the pairs (f, S) and (g, T) have a point of coincidence in X. Moreover, if the pairs (f, S) and (g, T) are weakly compatible, then f, g, S and T have a unique common fixed point.

Proof. The result follows from theorem 1, by taking as b-simulation function

$$\zeta(t, s) = \lambda s - t,$$

for all $t, s \geq 0$. □

Corollary 2. *Let (X, d) be a b-metric space and $f, T : X \to X$ be mappings such that*

$$\zeta(s^4 d(fx, fy), M(x, y)) \geq 0,$$

where

$$M(x, y) = \max\left\{d(Tx, Ty), d(fx, Tx), d(fy, Ty), \frac{d(fx, Ty) + d(Tx, fy)}{2s}\right\}.$$

Suppose that one of the pairs (f, T) satisfy the b-$(E.A)$-property and $T(X)$ is closed in X.

Then the pairs (f, T) has a point of coincidence in X. Moreover, if the pair (f, T) is weakly compatible, then f and T have a unique common fixed point.

Proof. Follows from Theorem 1 by choosing $f = g$ and $T = S$. □

Corollary 3. *Let (X, d) be a b-metric space and $f, T : X \to X$ be mappings, Suppose that there exists $\lambda \in [0, 1)$ such that*

$$s^4 d(fx, fy) \leq \lambda M(x, y),$$

where

$$M(x, y) = max\left\{d(Tx, Ty), d(fx, Tx), d(fy, Ty), \frac{d(fx, Ty) + d(Tx, fy)}{2s}\right\}.$$

Suppose that one of the pairs (f, T) satisfy the b-$(E.A)$-property and $T(X)$ is closed in X. Then the pairs (f, T) has a point of coincidence in X. Moreover, if the pair (f, T) is weakly compatible, then f and T have a unique common fixed point.

Proof. The result follows from corollary 2 , by taking as b-simulation function

$$\zeta(t, s) = \lambda s - t,$$

for all $t, s \geq 0$. □

The following is an example in support of our Theorem.

Example 4. Let $\zeta(t,s) = \dfrac{99}{100}s - t$, for all s,t in $[0,1)$, $X = [0,1]$ and define $d : X \times X \to [0;1)$ as follows

$$d(x,y) = \begin{cases} 0, & x = 0; \\ (x+y)^2, & x \neq y. \end{cases}$$

Then (X,d) is a b-metric space with constant $s = 2$. Let $f,g,S,T : X \to X$ be defined by

$$f(x) = x^2, \quad g(x) = \begin{cases} 0, & 0 \leq x < \frac{1}{4}, \\ \frac{1}{16}, & \frac{1}{4} \leq x < 1, \end{cases}$$
$$T(x) = x, \quad S(x) = \begin{cases} \frac{x}{2}, & 0 \leq x < \frac{1}{4}, \\ \frac{1}{16}, & \frac{1}{4} \leq x < 1. \end{cases}$$

Clearly, $f(X)$ is closed and $f(X) \subset T(X)$ and $g(X) \subset S(X)$. The sequence $\{x_n\}$; $x_n = \dfrac{1}{4} + \dfrac{1}{n}$, is in X such that $\lim\limits_{n\to\infty} fx_n = \lim\limits_{n\to\infty} Sx_n = \dfrac{1}{16}$. So that the pair (f,S) satisfies the $b-(E,A)$-property. To check the contractive condition (1), for all $x,y \in X$ if $x = y = 0$ or $x = y = \dfrac{1}{4}$, then (1) is satisfied.

If $x,y \in \left(0, \frac{1}{4}\right)$, then

$$d(fx,gy) = \left(x^2+0\right)^2 = x^4, \qquad d(fx,Sx) = \left(x^2 + \frac{x}{2}\right)^2,$$
$$d(Sx,Ty) = \left(\frac{x}{2} + x\right)^2 = \frac{9}{4}x^2, \quad d(gy,Ty) = (0+x)^2 = x^2,$$
$$d(fx,Ty) = \left(x^2+x\right)^2, \qquad d(Sx,gy) = \left(\frac{x}{2}+0\right)^2 = \frac{1}{4}x^2.$$

Now, we consider

$$\zeta(s^4 d(fx,gy), M(x,y)) = \frac{99}{100}M(x,y) - s^4 d(fx,gy) \geq 0,$$

so,

$$\frac{99}{100}M(x,y) - s^4 d(fx,gy) = \frac{99}{100} \cdot \frac{9}{4}x^2 - s^4 x^4 \geq 0.$$

If $x,y \in \left(\frac{1}{4}, 1\right)$, then

$$d(fx,gy) = \left(x^2 + \frac{1}{16}\right)^2, \quad d(fx,Sx) = \left(x^2 + \frac{1}{16}\right)^2,$$
$$d(Sx,Ty) = \left(\frac{1}{16} + x\right)^2, \quad d(gy,Ty) = \left(\frac{1}{16} + x\right)^2,$$
$$d(fx,Ty) = \left(x^2+x\right)^2, \qquad d(Sx,gy) = \left(\frac{1}{16} + \frac{1}{16}\right)^2 = \frac{1}{64}.$$

Thus,

$$\frac{99}{100}M(x,y)-s^4d(fx,gy)=\frac{99}{100}\left(x+\frac{1}{16}\right)^2-s^4\left(\frac{1}{16}+x^2\right)^2\geq 0.$$

As above results, we can find that the other cases are the same. Then (2.1) is satisfied for all $x,y\in X$, the pairs (f,S) and (g,T) are weakly compatible. Hence, all of the conditions of Theorem 2.1 are satisfied. and $x=0$ is the unique common fixed point of f, g,T and S.

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